# Bénard-Marangoni Instability in a Two-Layer System with Uniform Heat Flux

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**The onset of motion in two immiscible fluids heated from below by a uniform heat flux is studied analytically. The solution is based on the parallel-flow assumption. The critical Rayleigh and Marangoni numbers are determined. The superimposed cell mode in a single fluid layer is found to be possible when $Ma > 180$. The secondary cell in the two-layer system, if any, can only appear in one of the layers, and the conditions leading to the secondary cell are specified. The influence of upper free boundary, responses in a solid-layer and a fluid-layer system, as well as the use of equivalent Rayleigh numbers are also discussed.**

## Nomenclature

$a_1,a_2$ = constants in Eqs. (35) and (36)
$b_2$ = constant in Eq. (39)
$C$ = temperature gradient in the $x$ direction
$g$ = gravitational acceleration
$h$ = dimensionless height of layer 1, $H_1/H$
$H$ = height of channel
$H_i$ = height of layer $i$
$I,J,K$ = constants in Eqs. (41–43)
$k$ = thermal conductivity
$J(f,g)$ = Jacobian $f_x g_y - f_y g_x$
$L$ = length of cavity
$Ma$ = Marangoni number
$p$ = static pressure
$Pr$ = Prandtl number, $\mu/(\rho\alpha)$
$q$ = surface heat flux
$Ra$ = Rayleigh number, $(g\beta H^3/\nu\alpha)(qH/k)$
$T$ = temperature
$S$ = surface-tension gradient with respect to temperature
$u,v$ = velocity components in the $x$ and $y$ directions, respectively
$x,y$ = Cartesian coordinates
$\alpha$ = thermal diffusivity
$\beta$ = volumetric expansion coefficient
$\theta$ = dimensionless temperature varying with $y$
$\mu$ = dynamic viscosity
$\nu$ = kinematic viscosity
$\rho$ = density
$\sigma$ = surface tension
$\psi$ = stream function

*Subscripts*

$cr$ = critical quantities
$i$ = in the fluid layer $i$ ($i = 1,2$)
$r$ = relative quantities (layer 2 to layer 1)
$t$ = time derivative
$'$ = spatial derivative

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## Introduction

A HORIZONTAL layer heated from below has consistently attracted great attention in the last several decades. The interest stems from many important applications in chemical processes and geological and astrophysical systems, as well as by its theoretical importance in the understanding of flow bifurcation, nonlinear supercritical phenomena, and transition to chaos. It is now well-known that the fluid layer heated from below in the rest state will become unstable once the temperature gradient applied to it reaches a critical threshold. This is generally referred to as the Rayleigh-Bénard instability, which is a thermogravitational effect induced by the temperature gradient in a fluid with positive thermal expansion coefficient. Another instability known as Marangoni instability may come into play when the upper surface of the layer is free, or when another layer is superimposed on it. This Marangoni instability is a thermocapillary effect which is due to the temperature dependence of surface tension at the interfaces. Most studies on Bénard-Marangoni convection deal with single layers with an upper free surface.[1-5] There is also the study related to two-layer instability by Catton and Lienhard[6] in which an interlayer solid of finite thermal conductivity is inserted between the layers so that no thermocapillary effect is considered.

This paper is aimed at studying the onset of motion of Bénard-Marangoni convection in a two-layer system. The problem consists of a shallow rectangular cavity with steady uniform heat flux through the bottom and the top, the two far end walls being adiabatic (as shown in Fig. 1). The approximation of parallel flows in each layer enables the stream function and temperature to be expressed as fourth-order and fifth-order polynomials, respectively. By properly setting the boundary conditions at the upper and lower boundaries and also at the layer interface, the unknown coefficients in the polynomials can be determined. With energy conservation

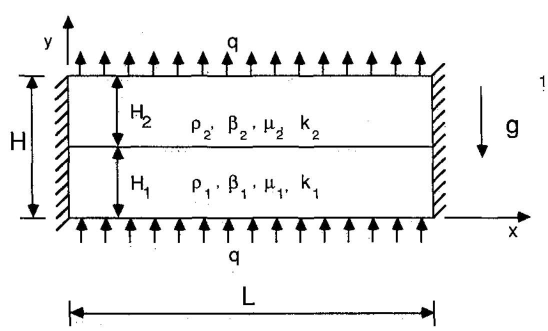


**Fig. 1 Two-fluid-layer system.**

across each horizontal plane, the critical Rayleigh and Marangoni numbers for the onset of motion can be obtained. As a first step, the study is on a two-layer system with thermocapillary forces acting at the interface. Attention is drawn to the multiple steady states, stability of multicells for a single layer, stability of the system under microgravitational conditions, the combined effect of the thermogravitational and thermocapillary forces, the physical significance of the critical Rayleigh number, and also the influence of the relative thickness of each layer.

## Governing Equations and Boundary Conditions

A two-dimensionless shallow cavity heated from below with uniform heat flux $q$ and end walls being adiabatic as depicted in Fig. 1 is considered. The cavity is filled with two different, viscous, immiscible fluids with dimensions $L \times H_1$ and $L \times H_2$ at the rest state, respectively. Let subscript $i$ denote the quantities in the fluid $i$ ($i = 1,2$). With the usual Boussinesq approximation and the approximation of constant physical properties for small temperature differences,[7] one can write the continuity equation, the Navier-Stokes equations, and the energy for the steady state as follows:

$$(u_i)_{,x} + (v_i)_{,y} = 0 \tag{1}$$

$$u_i(u_i)_{,x} + v_i(u_i)_{,y} = -1/\rho_i p_{i,x} + \nu_i[(u_i)_{,xx} + (u_i)_{,yy}] \tag{2}$$

$$u_i(v_i)_{,x} + v_i(v_i)_{,y} = -1/\rho_i p_{i,y} + \nu_i[(v_i)_{,xx} + (v_i)_{,yy}] + g\beta_i \Delta T \tag{3}$$

$$u_i(T_i)_{,x} + v_i(T_i)_{,y} = \alpha_i[(T_i)_{,xx} + (T_i)_{,yy}] \tag{4}$$

The boundary conditions at the solid walls are those of no-slip conditions for the velocity and constant temperature gradient (adiabatic or fixed heat flux) for the temperature:

$$y = 0, \quad u_1 = v_1 = 0, \quad -k_1(T_1)_{,y} = q \tag{5}$$

$$y = H, \quad u_2 = v_2 = 0, \quad -k_2(T_2)_{,y} = q \tag{6}$$

$$x = 0, L, \quad u_i = v_i = 0, \quad -k_i(T_i)_{,x} = 0 \tag{7}$$

At the interface, it is assumed that a plane surface holds for all times, an assumption which will greatly simplify the analysis. The continuities of temperature, heat flux, velocity, shear stress, and pressure require that,

$$y = H_1, \quad T_1 = T_2, \quad -k_1(T_1)_{,y} = -k_2(T_2)_{,y},$$
$$u_1 = u_2, \quad v_1 = v_2 = 0, \quad \mu_1[(u_1)_{,y}] = \mu_2[(u_2)_{,y}] - ST_{,x},$$
$$p_1 = p_2 \tag{8}$$

where $S = -(\sigma)_{,T}$.

A stream function is introduced so that the continuity equation is automatically satisfied:

$$u_i = (\psi_i)_{,y}, \quad v_i = -(\psi_i)_{,x} \tag{9}$$

To write the system of equations in dimensionless form, the following nomalizing quantities are used: $H$ for length, $\alpha_1/H$ for velocity, $qH/K_1$ for temperature, and $\alpha_1$ for the stream function. After some manipulations, Eqs. (1–4) become

$$J(\psi_1, \nabla^2\psi_1) = Pr_1 \nabla^4\psi_1 - Pr_1 Ra_1(T_1)_{,x} \tag{10}$$

$$J(\psi_2, \nabla^2\psi_2) = \alpha_r Pr_2 \nabla^4\psi_2 - \alpha_r^2 k_r Pr_2 Ra_2(T_2)_{,x} \tag{11}$$

$$J(\psi_1, T_1) = -\nabla^2 T_1 \tag{12}$$

$$J(\psi_2, T_2) = -\alpha_r \nabla^2 T_2 \tag{13}$$

where $J(f,g) = f_x g_y - f_y g_x$. The controlling parameters are

$$Ra_i = \frac{g\beta H^3}{\alpha_i \nu_i}\frac{qH}{k_i} \tag{14}$$

$$Pr_i = \frac{\nu_i}{\alpha_i} \tag{15}$$

and the physical property ratios are

$$\alpha_r = \frac{\alpha_2}{\alpha_1}, \quad \mu_r = \frac{\mu_2}{\mu_1}, \quad \rho_r = \frac{\rho_2}{\rho_1}, \quad k_r = \frac{k_2}{k_1} \tag{16}$$

The corresponding boundary conditions then become

$$y = 0, \quad \psi_1 = (\psi_1)_{,y} = 0, \quad -(T_1)_{,y} = 1 \tag{17}$$

$$y = 1, \quad \psi_2 = (\psi_2)_{,y} = 0, \quad -k_r(T_2)_{,y} = 1 \tag{18}$$

$$x = 0, L/H, \quad (\psi_i) = (\psi_i)_{,x} = 0, \quad (T_1)_{,x} = 0,$$
$$k_r(T_2)_{,x} = 0 \tag{19}$$

At the interface $y = h = H_1/H$,

$$T_1 = T_2, \quad (T_1)_{,y} = k_r(T_2)_{,y} \tag{20a}$$

$$\psi_1 = \psi_2 = 0, \quad (\psi_1)_{,y} = (\psi_2)_{,y} \tag{20b}$$

$$(\psi_1)_{,x} = (\psi_2)_{,x} = 0 \tag{20c}$$

$$(\psi_1)_{,yy} = \mu_r(\psi_2)_{,yy} - MaT_{,x} \tag{20d}$$

where $Ma$ is the Marangoni number, defined as

$$Ma = \frac{SH}{\alpha_1\mu_1}\frac{qH}{k_1} \tag{21}$$

## Analytical Solution

The direct solution of Eqs. (10–13) with boundary conditions Eqs. (17–20) is difficult. Previous solutions have been made by using the Galerkin method[6,8] as outlined by Chandrasekhar,[9] or by convergent power series.[10] Direct numerical solution of Eqs. (10–13) can be performed with extensive computations for the eigenvalue problems. The present study, however, seeks an analytical solution in closed form, which is possible under the assumption of parallel flows over a large portion of the layers. It is to be noted that the parallel flow is one of many possible flows starting from the rest state. Such parallel flow can be realized even for the uniform heat flux condition, since it is possible that cellular convection patterns in the top and bottom layers could adjust the temperature distributions in such a manner that constant heat flux conditions can be satisfied both at the top and the bottom walls, but not necessarily in the middle interface. On the other hand, this may also result in the appearance of transverse cells,[11] and thus make the parallel-flow assumption invalid. In any case, the parallel-flow assumption is held only in the region away from the end walls. The present paper is not to address all possible onset motions for the two-layer system, but to present results for one of the possible motions.

In the parallel-flow approximation, the flow is assumed to depend on $y$ only, so that $v = 0$, $u = u(y)$, and the temperature field is assumed to be a superposition of a linear function of $x$ and an unknown function of $y$. This approximation is originally from the study of infinitely extended thin layers, and has been used to study natural convection in a shallow cavity heated at the two ends by Cormak et al.[12] and by Bejan and Tien,[13] and in a shallow horizontal cylindrical cavity heated at the two ends by Bejan and Tien.[14] It has been extended to a shallow inclined cavity to study the multiple steady states of flow by Vasseur et al.[15] With this approximation, obviously the boundary conditions of Eq. (19) in the $x$ direction cannot be exactly satisfied; instead an integral condition on the average flux at any $y$ section is used as in the following:

$$\int_0^1 (uT - T_{,x})\,dy = 0 \tag{22}$$

For the uniform flux heating in a single layer as demonstrated in Ref. 15, the parallel-flow approximation gives a reasonable prediction on the flow and heat transfer for the cavity with aspect ratios less than 0.5 (shallow cavity). A uniform heat-flux condition, unlike the isothermal case, is more likely to satisfy the parallel-flow assumption. In the latter case, a multicellular structure may appear, leading to nonuniform fluxes.[16] With this approximation, we then have

$$\psi_i = \psi_i(y) \tag{23}$$

$$T_i = C_i x + \theta_i(y) \tag{24}$$

where $C_i$ are the unknown constant temperature gradients in the $x$ direction in the fluid layers. First, as the continuity of temperature at interface requires, it can be shown that at $y = h$ for any $x$, $T_1 = T_2$ will result in

$$C_1 = C_2 = C \tag{25}$$

With Eqs. (23-25), the governing equations can be simplified to

$$(\psi_1)_{,yyyy} = Ra_1 C \tag{26}$$

$$(\theta_1)_{,yy} = C(\psi_1)_{,y} \tag{27}$$

$$(\psi_2)_{,yyyy} = \alpha_r k_r Ra_2 C \tag{28}$$

$$\alpha_r(\theta_2)_{,yy} = C(\psi_2)_{,y} \tag{29}$$

and boundary conditions are simply,

$$y = 0, \qquad \psi_1 = (\psi_1)_{,y} = 0, \qquad -(\theta_1)_{,y} = 1 \tag{30}$$

$$y = 1, \qquad \psi_2 = (\psi_2)_{,y} = 0, \qquad -k_r(\theta_2)_{,y} = 1 \tag{31}$$

and at the interface $y = h = H_1/H$

$$\theta_1 = \theta_2, \qquad (\theta_1)_{,y} = k_r(\theta_2)_{,y}$$

$$\psi_1 = \psi_2 = 0, \qquad (\psi_1)_{,y} = (\psi_2)_{,y} \tag{32}$$

$$(\psi_1)_{,yy}\mu_r(\psi_2)_{,yy} - MaC$$

Solutions of Eqs. (26) and (28) are fourth-order polynomials. With boundary conditions Eqs. (30) and (31), and the conditions of $\psi_1 = \psi_2 = 0$ at $y = h$, we have

$$\psi_1 = Ra_1 C/24\, y^2(y^2 - h^2) + a_1 C y^2(y - h) \tag{33}$$

$$\psi_2 = \alpha_r k_r Ra_2 C/24(1 - y)^2[(1 - y)^2 - (1 - h)^2]$$
$$+ a_2 C(1 - y)^2[(1 - y) - (1 - h)] \tag{34}$$

Here $a_1$ and $a_2$ are constants. With the continuity of velocity and shear stress, $a_1$ and $a_2$ can be determined as

$$\alpha_1 = \frac{\mu_r \alpha_r k_r Ra_2(1-h)^3 - Ra_1 h^2[4\mu_r h + 5(1-h)] - 12Ma(1-h)}{48h(1-h+\mu_r h)} \tag{35}$$

$$\alpha_2 = \frac{Ra_1 h^3 - \alpha_r k_r Ra_2(1-h)^2[4(1-h) + 5\mu_r h] + 12Ma\, h}{48(1-h)(1-h+\mu_r h)} \tag{36}$$

Solutions to Eqs. (27) and (29) can be written as

$$\theta_1 = Ra_1 C/24(y^5/5 - y^3h^2/3) + a_1 C^2(y^4/4 - y^3h/3) - y \tag{37}$$

$$\theta_2 = k_r Ra^2 C^2/24\,[-(1-y)^5/5 + (1-y)^3(1-h)^2/3]$$
$$+ a_2/\alpha_r C^2\,[-(1-y)^4/4 + (1-y)^3(1-h)/3] - y/k_r + b_2 \tag{38}$$

where

$$b_2 = -C^2/180\,[Ra_1 h^5 + k_r Ra_2(1-h)^5] - C^2/12$$
$$[a_1 h^4 + (a_2/\alpha_r)(1-h)^4] - h + h/k_r \tag{39}$$

To determine the constant $C$, which is the temperature gradient in the $x$ direction, the integral equation given by Eq. (22) is used, which represents the local zero net heat flux in the $x$ direction. By substituting all of the quantities, we have

$$KC^3 - (I - J)C = 0 \tag{40}$$

Here $K$, $I$, and $J$ are constants given by

$$K = \int_0^h (\psi_1/C)^2\,dy + \int_h^1 k_r/\alpha_r(\psi_2/C)^2\,dy \tag{41}$$

$$I = \int_0^h (\psi_1/C)\,dy + \int_h^1 (\psi_2/C)\,dy \tag{42}$$

$$J = h + (1-h)k_r \tag{43}$$

Solutions for $C$ are then

$$C = \begin{cases} 0 \\ \pm(I-J)^{1/2}/K^{1/2} \end{cases} \tag{44}$$

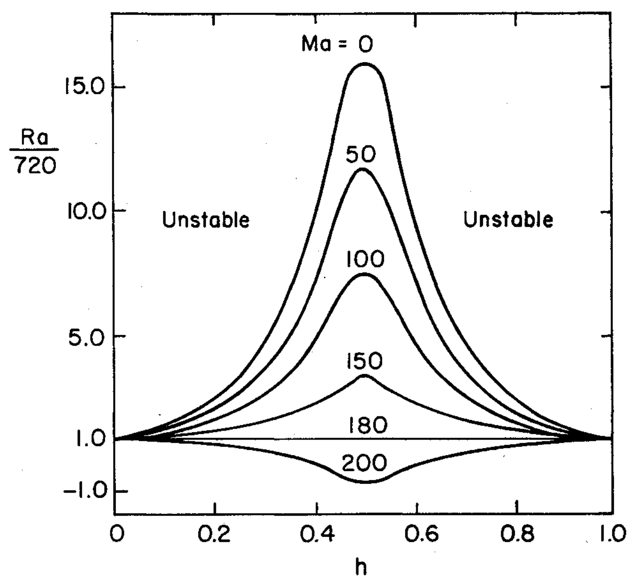


**Fig. 2 Marginal-stability curves for a single fluid layer at different Marangoni numbers.**

Since $K$ is always positive, $C=0$, in the case of $I<J$, is the only real root of $C$; i.e., there is no convection. If $I>J$, two sets of convection cells bifurcate from the rest state [as indicated in Eq. (44) with both positive and negative $C$]. The marginal state, which determines the critical Rayleigh number and critical Marangoni number, is when $I=J$, that is

$$Ra_1h^5 + Ra_2k_r(1-h)^5 + \frac{5h(1-h)[\alpha_r h^2 - (1-h)^2][Ra_1h^2 - Ra_2\alpha_r\mu_r k_r(1-h)^2 + 12Ma]}{4\alpha_r(1-h+\mu_r h)} = 720\,[h + (1-h)k_r] \quad (45)$$

It is noted here that $I$ and $J$ in Eq. (44) represent an equivalent thermal conductivity for cases with and without motion, respectively. When $J$ is large compared to $I$, the heat is transferred largely by conduction, and convection dominates when $J$ is small compared to $I$. The case $I=J$ represents an equilibrium between the two effects.

## Results and Discussions

Equation (45) is a general expression for the onset of motion of two superimposed fluid layers. Several interesting cases are possible and can be physically discussed as follows:

### Stability of Two Superimposed Cell Mode in a Single Fluid

One of the important aspects of the above criterion is the stability of two superimposed cell mode in a single fluid. Onset of motion of a single cell mode in a fluid layer with uniform heat flux has been studied previously, and it has been shown that the critical Rayleigh number is 720.[11] This result can be recovered from Eq. (45) by setting $h=1.0$. It remains to be seen relative to the possible onset of motion of the two-cell mode. To do so, let $u_r=\alpha_r=k_r=1.0$, and $Ra_2=Ra_1$ for the single-layer case, and then Eq. (45) reduces to

$$Ra_1\,[1 - 15/4h(1-h)] + 15h(1-h)Ma - 720 = 0 \quad (46)$$

Figure 2 shows the marginal-stability curve for a single fluid at different Marangoni numbers with independent variable $h$. It can be seen that the Marangoni number has a critical influence on the shape of the convective motion of the fluid. When $Ma<180$, the convective motion is in the form of a unicell with a critical Rayleigh number of 720. When $Ma>180$, the motion is always in a two-cell mode for any positive Rayleigh number. Displayed in Fig. 3 is the critical Marangoni number curve at microgravitational condition ($Ra_1\to 0$). The critical Marangoni number is 192, and the convective motion is in a two-cell mode. Another interesting phenomenon is that, without the Marangoni effect, the two-cell mode with equal size is possible only if the Rayleigh number is 16 times higher than the one for the convective motion of a single mode. It agrees with the results of Catton and Lienhard,[6] who considered a two-fluid-layer system with isothermal boundary conditions and a solid interlayer.

### Single Layer Exposed to a Free Surface

Most studies on Bénard-Marangoni convection are on the thermocapillary flow developed in a horizontal layer heated from below when its upper boundary is a free surface. In this case, the Marangoni force plays an important role in the initiation of motion of the fluid. This situation can be recovered from Eq. (45) by letting $u_r$ approach zero and $h$ approach unity. This gives

$$Ma/48 + Ra_1/320 = 1 \quad (47)$$

which is exactly the one obtained by Garcia-Ybarra et al.[5] using a linear stability analysis. The stability curve is shown in Fig. 4. When the Marangoni effect is negligibly small, this results in a threshold value for buoyancy-driven single layer with an upper free surface of $R_{cr}=320$, which is the same as that obtained by Sparrow et al.[11] Under microgravitational condition, the motion at the onset corresponds to a threshold value for the Marangoni number of $Ma_{cr}=48$. In both limiting cases, the critical values ($R_{cr}$ and $Ma_{cr}$) are much smaller than those with rigid upper surface (720 and 192, respectively).

### Flow Structure

The velocity profile inside fluid layers can take various forms as schematically indicated in Fig. 5. Of particular interest is the appearance of a secondary cell in a one-fluid layer. This phenomenon has been found for two-layer systems with horizontal heating.[17-19] In several instances, the secondary cell is so small that it is difficult to resolve them numerically. To this end, we will discuss the condition that leads to the secondary cells and their positions. One obvious reason for the appearance of the secondary cell is that there exists a zero value for $\psi_1$ at $y_1(\psi_1=0)$, which satisfies $0<y_1<h$, and for $\psi_2$

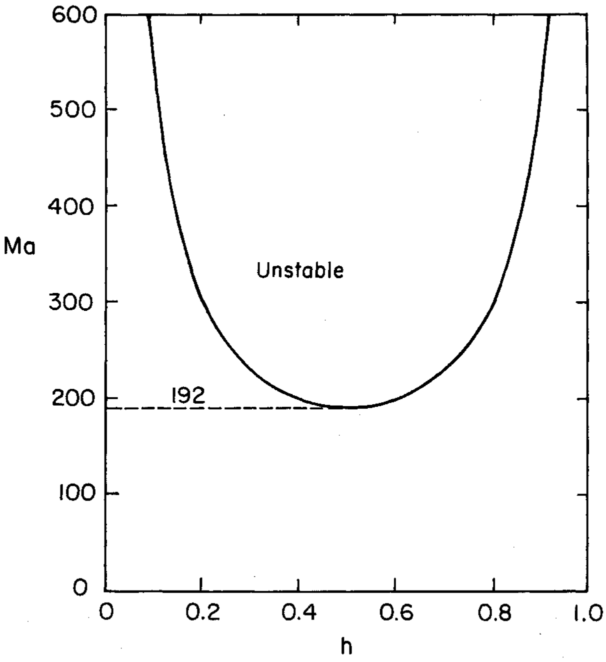


**Fig. 3 Critical Marangoni number at microgravitational condition.**

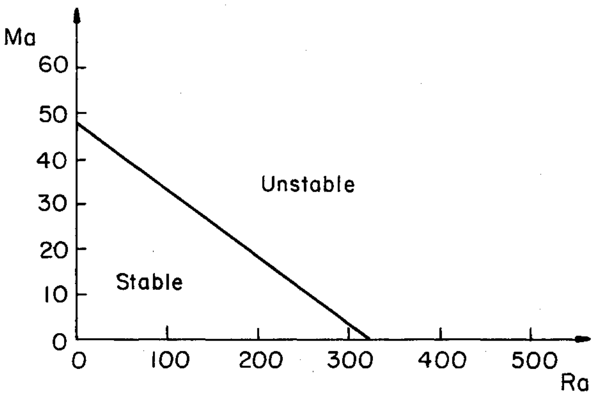


**Fig. 4 Stability curve for a single layer with upper free surface.**

at $y_2(\psi_2 = 0)$, which satisfies $h < y_2 < 1$. These $y$s can be easily found as

$$y_1(\psi_1 = 0) = \frac{Ra_1 h^2 [2\mu_r h + 3(1-h)] - \mu_r \alpha_r k_r Ra_2 (1-h)^3 + 12Ma(1-h)}{2Ra_1 h\ [(1-h) + \mu_r h]} \tag{48}$$

$$1 - y_2(\psi_2 = 0) = \frac{\alpha_r k_r Ra_2 (1-h)^2 [2(1-h) + 3\mu_r h] - Ra_1 h^3 - 12Ma\ h}{2\alpha_r Ra_2 k_r (1-h)\ [(1-h) + \mu_r h]} \tag{49}$$

The condition $(y_1 - h) < 0$ gives

$$Ra_1 h^2 - \mu_r \alpha_r k_r R_{a2}(1-h)^2 + 12Ma < 0 \tag{50}$$

and similarily, the condition $[(1-y_2) - (1-h)] < 0$ gives

$$-[Ra_1 h^2 - \mu_r \alpha_r k_r\ Ra_2 (1-h)^2 + 12Ma] < 0 \tag{51}$$

The above two conditions are contradictory in that they cannot be satisfied at the same time. This leads to the conclusion that a secondary cell, if any, can only appear in one of the fluid layers. The condition of Eq. (50) can also be written as

$$Ra_1 [\beta_1 h^2 - \beta_2 (1-h)^2] + 12Ma < 0 \tag{52}$$

It is apparent that both Marangoni force and thermal expansion are responsible for the appearance of the secondary cell in the fluid layers.

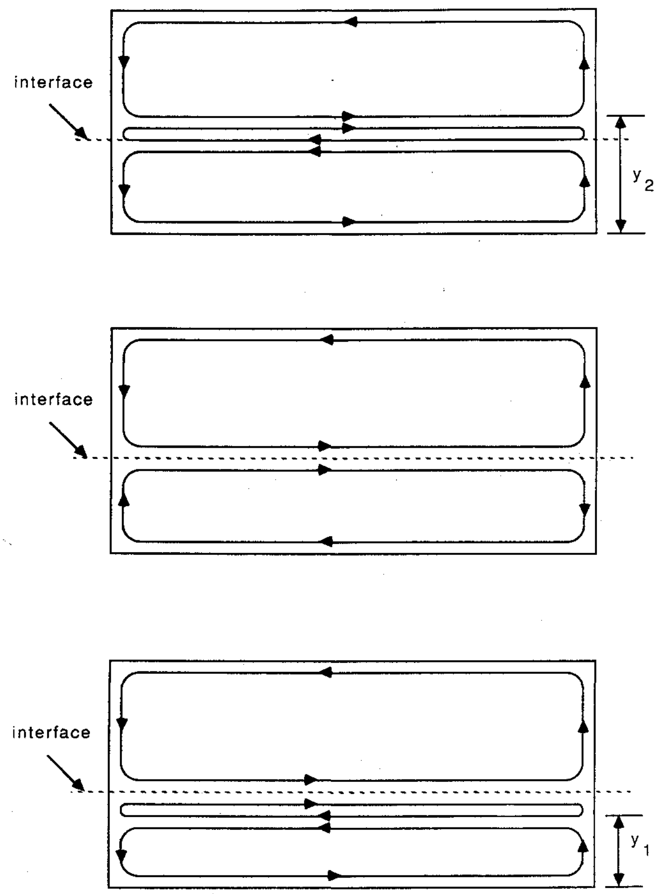


Fig. 5 Flow strucures in a two-layer system.

#### Equivalent Rayleigh Number for Two Layers

In the two-layer system, it is always desirable to consider an overall equivalent Rayleigh number and consequently an overall critical Rayleigh number. Since the Rayleigh number consists of a length scale and the thermophysical properties of $k$, $\mu$, $\alpha$, and $\beta$, it is interesting to find out what combination of them will occur. When there is no secondary cell existing in any layer, or when

$$Ra_1\ [\beta_1 h^2 - \beta_2 (1-h)^2] + 12Ma = 0 \tag{53}$$

we have from Eq. (45)

$$Ra_{1(h)} h + Ra_{2(1-h)} k_r (1-h) = 720\ [h + (1-h) k_r] \tag{54}$$

where subscripts $(h)$ and $(1-h)$ denote the length scales that the Rayleigh numbers are based on. Evidently, the overall equivalent Rayleigh number can be defined as the weighted average of the Rayleigh numbers in the two individual layers. This average is based on the product of the thermal conductivity and the height of the layer. However, when Eq. (53) is not satisfied, there is no simple expression for the overall equivalent Rayleigh number.

#### Instability of Superposed Solid and Fluid Layers

The system consisting of a solid layer and a liquid layer is examined next. The problem can be analyzed by letting $\mu_r \to \infty$ and $Ra_2 = 0$; thus, marginal stability is given by

$$Rah^5 - 720\ [h + (1-h) k_r] = 0 \tag{55}$$

or

$$Ra_{cr} = 720\ [h + (1-h) k_r]/h^5 \tag{56}$$

If the Rayleigh number is defined in terms of the fluid layer thickness,

$$Ra_{cr(h)} = 720\ [h + (1-h) k_r]/h \tag{57}$$

The critical Rayleigh number thus varies with the thermal conductivity of the solid. When $k_r = 0$, it is the same as for a single layer, i.e., $Ra_{cr} = 720$. However, with increasing $k_r$, $Ra_{cr}$ increases linearly. This is due to the distribution of thermal resistance in each layer.

## Conclusions

The solution of the onset of motion of Bénard-Marangoni convection for a two-layer system in a shallow cavity with uniform wall heat flux is derived. The limiting cases of a single layer with an upper rigid and free surfaces can be recovered, and these compare well with known results. Despite the relative simplicity of the final solution, it does show some basic features that can easily be realized and analyzed without going through extensive computational efforts. The following conclusions can be made:

1) The convective motion of the two superimposed cell mode in a single fluid depends on the Marangoni number. At $Ma > 180$, it is in a two-cell mode with equal size. Without the Marangoni effect, the critical Rayleigh number for the two-cell mode is at least 16 times higher than that for a single-cell mode.

2) The upper boundary condition has a pronounced influence on the critical Rayleigh and Marangoni numbers for a single fluid. A free upper boundary will result in much smaller critical values.

3) The appearance of a secondary cell in a fluid layer depends on the thermal expansion coefficients and the Marangoni number, as well as on the relative thickness of the fluids. It can only be generated in one of the layers. The conditions leading to the secondary cell are specified.

4) Without the secondary cell in any layer, an overall equivalent Rayleigh number for a system can be defined as an average of the Rayleigh numbers in each layer weighted by the product of thermal conductivity and the height of the layer.

5) When the system consists of a solid and a fluid layer, the conductivity of the solid has an important effect on the critical Rayleigh number, which increases linearly with that of the conductivity rato of the solid and the fluid.

As pointed out earlier, the present analysis and results are limited to the parallel-flow assumption. Other transitions are also possible, and further results will be reported in the future.

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